An Adaptive Fixed Point Iteration Algorithm for Finite Element Analysis with Magnetic Hysteresis Materials

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In this paper, an adaptive fixed point iteration algorithm is proposed to solve magnetic field problems with magnetic hysteresis. The iteration starts with the *B*-correction scheme. If the solution is not converged to a given accuracy after a certain number of iterations, the *H*-correction scheme is used to continue the iteration. During the whole iteration process, the scheme with minimum error is recorded, and it is used for the next time step to start a new iteration process. We always try to find the best scheme if the given accuracy is not achieved after a certain number of iterations. The application examples show that the proposed algorithm not only has very fast convergence rate, but is also very stable.

***Index Terms*—Finite element methods, fixed point method, hysteresis, magnetic materials.**

I. INTRODUCTION

FOR a time-stepping transient magnetic field problem with hysteresis media involved, Raphson (NR) method is not directly applicable because the derivative dB/dH at the last field solution is not well defined due to two different values in two field changing (increasing and decreasing) directions. Instead, the fixed point method plays an important role in the iteration of nonlinear problems with hysteresis.

There are two schemes in the fixed point method for magnetic field computation, the *B*-correction and *H*-correction schemes. The former uses curve $H(B)$, and the later uses $B(H)$, during the iteration. There are two basic methods for each scheme, the global-coefficient method and local-coefficient method. In the global-coefficient method with *B*-correction scheme, the magnetic reluctivity is set to the average value of the minimum and maximum differential reluctivities, and keeps constant for all elements in all time steps [1]. This method is stable for a wide range of starting point, but it suffers from slow convergence.

In the local-coefficient method, the magnetic reluctivity is different for each mesh element at each time step, but it keeps constant during the iteration [2]-[5]. The constant reluctivity is set to be the differential reluctivity of the previous time step, multiplying by a global convergence factor which is greater than one [2]-[3]. The optimal value of the convergence factor is found by linear search. If the starting point is sufficiently close to the fixed point, this method will significantly speed up the convergence rate. However, convergence from any starting point is no longer guaranteed [4].

In this paper, an adaptive fixed point iteration algorithm is proposed. The iteration starts with the *B*-correction scheme in which the constant reluctivity is set to the maximum differential reluctivity. If the solution is not converged to a given accuracy after a certain number of iterations, the iteration will be continued by switching to the *H*-correction scheme in which the constant permeability is set to the maximum differential permeability. The two schemes are alternately used to keep minimum number of iterations. The application examples show that the proposed algorithm not only has very fast convergence rate, but is also very stable.

II. FIXED POINT METHOD

For any continuous nonlinear function $y = f(x)$, if y is known as y_0, the equation can be rewritten as $x = F(x)$. The root of the equation can be computed by the fixed point iteration as

$$x_{k+1} = F(x_k), \quad k = 0, 1, 2, ... \tag{1}$$

For any period $[a, b]$, if

$$\left|F(b) - F(a)\right| \le L\left|b - a\right| \tag{2}$$

the iteration of (1) will converge to a fixed point, as long as $L<1$.

To better appreciate the fixed point iteration algorithm, let's take an inductor with uniform cross-section core excited by a coil of N turns carrying a current of $i(t)$ as an illustration example. If the core is treated as a one-dimensional element and the magnetic property is expressed as $H(B)$, the fixed point iteration in the *B*-correction scheme is.

$$B_{k+1} = B_k + \frac{H_a - H(B_k)}{\nu_{FP}} = F(B_k) \tag{3}$$

where $H_a = Ni(t)/l$ is constant during the iteration with l being the average length of the core. The constant reluctivity ν_{FP} can freely be selected provided that (2) is satisfied.

If the constant reluctivity is selected as

$$\nu_{FP} = \nu_{d\max} = 1/\mu_0 \tag{4}$$

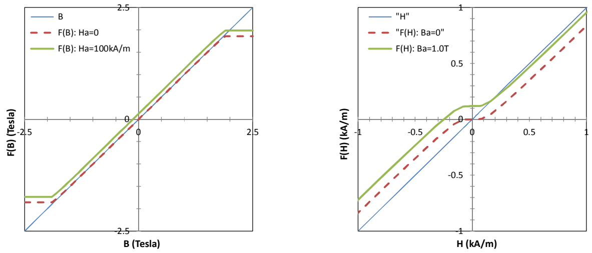
the iteration will converge very fast when the fixed point is in the saturated region where the slope L is close to 0, see Fig. 1(a). When the fixed point is in the unsaturated region, convergence is also guaranteed, but with slow convergence rate because L is close to 1.0.

When magnetic property is expressed as $B(H)$, the fixed point iteration using *H*-correction scheme is expressed as

$$H_{k+1} = H_k + \frac{B_a - B(H_k)}{\mu_{FP}} = F(H_k) \tag{5}$$

where B_a is the applied flux density which is constant during the iteration. The constant permeability μ_{FP} can be selected as

$$\mu_{FP} = \mu_{d\max} \tag{6}$$



(a) (b)

Fig. 1 Iterative function: (a) *B*-correction method based on (3)-(4); (b) *H*-correction method based on (5)-(6).

where $\mu_{d\max}$ is the maximum differential permeability of the curve $B(H)$. In such a case, the convergence is guaranteed. However, the performance using the *H*-correction scheme is different between in the saturated region with fast convergence rate and in the unsaturated region with very slow convergence rate, as indicated in Fig. 1(b).

III. Adaptive Fixed Point Algorithm

As discussed above, the convergence behavior using different schemes is different in different regions. Therefore, we propose an adaptive fixed point iteration algorithm. The iteration may start with the *B*-correction scheme in which the constant reluctivity is set to the maximum differential reluctivity. If the solution is not converged to a given accuracy after a certain preset number of iterations, the iteration will be continued by switching to the *H*-correction scheme in which the constant permeability is set to the maximum differential permeability. With the combined use of the two correction schemes during the entire iteration process, the scheme type with less number of iterations will be recorded and used as the initial scheme type for the next time step. The flowchart of the iteration process is shown in Fig. 2. The propose approach is applicable because: 1) the intermediate field solution *B* and *H* can be obtained from each other based on the identified *BH* relationship of the previous iteration; 2) in terms of the used hysteresis model [6], *B* and *H* can be derived from each other.

IV. Validations

To validate the effectiveness of the proposed algorithm, an inductor is taken as a benchmark to compare the average iteration numbers among different iteration types. Table I shows the average iteration number using the proposed adaptive algorithm implemented 2D transient solver compared with those using either *B*-correction or *H*-correction scheme under sinusoidal current and voltage excitations respectively.

Table I. Comparison of Average Iteration Number among Different Iteration Types

Iteration type	Source type	Average iteration number	Total time steps	Non-converged time steps
B-correction scheme	Current	96.47	100	6
	Voltage	23.23	100	0
H-correction scheme	Current	37.77	100	0
	Voltage	124.82	100	5
Adaptive algorithm	Current	37.58	100	0
	Voltage	23.23	100	0

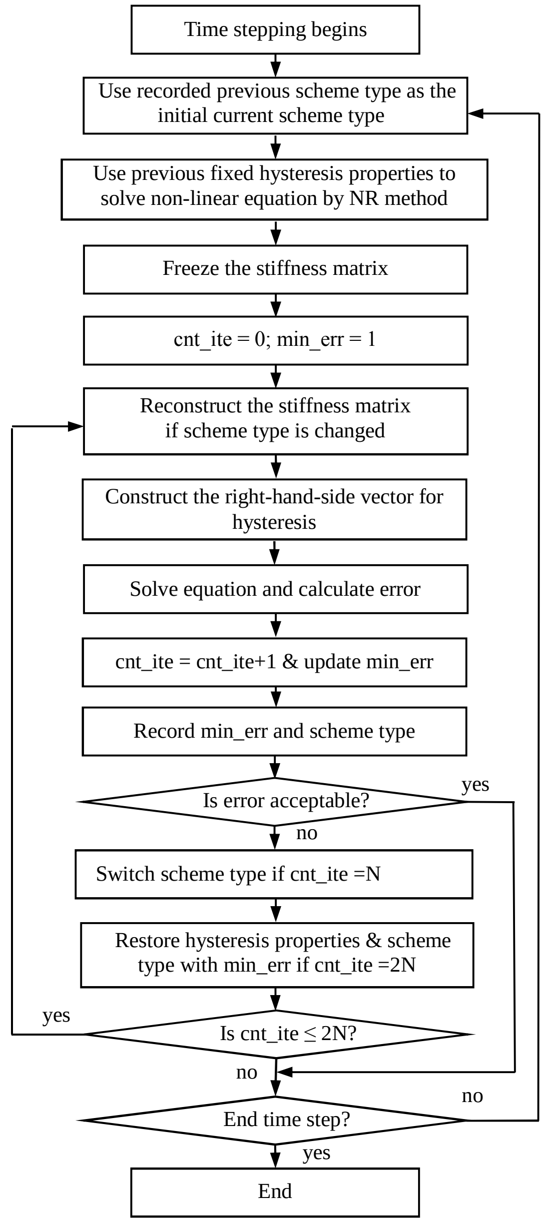


Fig. 2 The flowchart for the proposed adaptive fixed point iteration

V. References

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